APPLICATION OF PYTHON PROGRAMMING TOOLS FOR CRITICALITY SIMULATION OF NEUTRON TRANSPORT IN NUCLEAR REACTOR WITH SLAB GEOMETRY

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ABSTRACT

Monte Carlo criticality calculations use the power iteration method to determine the eigenvalue (k_{eff}) and eigenfunction (fission source distribution) of the fundamental mode. However, the main problems of this method are the slow convergence of fission source distribution from the initial guess to the stationary solution, and the correlation between successive cycles which results in an under-prediction bias in the confident intervals of the estimated response. In this paper, we presented the Wielandt's method aiming to accelerate the convergence of the Monte Carlo power iteration. The object-oriented programming called Python prototype, was used to describe the standard Monte Carlo criticality power iterations for mono-kinetic particles and to compare the results obtained by the two different methods of acceleration mentioned above. The Wielandt's method successfully suppressed the auto-correlation, even though no gain in the figure of merit seemed to occur.

Keywords: Eigenvalue, eigenfunction, Monte Carlo criticality, power iteration, Wielandt's.

Ứng dụng ngôn ngữ lập trình hướng đối tượng python cho bài toán mô phỏng sự truyền neutron trong lò hạt nhân trong điều kiện tới hạn với cấu trúc hình học phẳng

TÓM TẮT

Sử dụng phương pháp lặp theo hàm mũ để giải phương trình truyền neutron trong lò phản ứng hạt nhân với cấu trúc hình học phẳng, một chiều bằng phương pháp mô phỏng Monte Carlo để xác định trị riêng và hàm riêng (tương ứng với sự phân bố của nguồn phân hạch) tồn tại hai vấn đề chính là: (1) sự hội tụ chậm của nguồn phân hạch dự đoán ban đầu về phân bố thực; (2) mối liên hệ giữa các chu kỳ trong mỗi mô phỏng sẽ dẫn đến sai số khi đánh giá độ tin cậy của kết quả thu được. Do đó, nghiên cứu này đã đề cập tới một phương pháp mới, Wielandt, với mục đích tăng tốc quá trình hội tụ của nguồn ban đầu và đưa ra phương án mới để đánh giá mối liên hệ giữa các chu kì liên tiếp trong một mô phỏng độc lập. Ngôn ngữ lập trình hướng đối tượng có tên gọi Python đã được sử dụng để mô tả sự truyền của hạt neutron đơn năng, đồng thời so sánh kết quả thu được bằng hai phương pháp khác nhau. Kết quả cho thấy, phương pháp mới đã giải quyết thành công các vấn đề đặt ra của bài toán Monte Carlo. Tuy nhiên, bài toán về hệ số phẩm chất vẫn cần tiếp tục nghiên cứu thêm.

Từ khóa: Bài toán tới hạn Monte Carlo, hàm riêng, lặp theo hàm số mũ, phương pháp Wielandt, trị riêng.

1. INTRODUCTION

Basically, the behaviour of nuclear reactor can be simulated by coupling neutron transport theory and thermal-hydraulics. The neutron transport theory studies the interaction of neutron with matter, which is described by the Boltzmann equation (Reuss, 2008). Therefore, the computer codes developed for nuclear reactor simulations have to solve this equation in order to accurately describe the neutron transport in the reactor. Nowadays, the development of powerful computer has enhanced the capability of not only

deterministic codes to numerically solve the neutron transport equation, but also of stochastic codes with very detailed geometry and continuous energy description of the neutron collisions. These stochastic codes are known as Monte Carlo codes.

The basic principle of Monte Carlo method in particle transport is to simulate the entire life of each particle from its initial emission until its death either by absorption or leakage from the system boundaries. The frequency, nature and outcome of every interaction that may occur during the particle life are randomly sampled according to algorithms derived from particle physics laws. When the process is repeated for a large number of particles, the averages of the obtained results yield a detailed description of the transport process. However, the main problem in Monte Carlo criticality calculations is the convergence of the power iteration (https://docs.python.org/2/tutorial/) and the technique used to converge an arbitrary initial guess source distribution to thecritical source distribution. After the source is converged, it is possible to tally desired quantities like the dominant eigenvalue of the Boltzmann equationk_{eff}, the corresponding eigenvector (the stationary flux distribution), reaction rates, etc.

This convergence is known to be problematic when the dominante ratio (ratio between the second and the first eigenvalues of the system) is close to one. This undesired phenomenon occurs for large systems of the size of about 100 times the neutron mean free path. The consequence of the power iteration is, on one hand, the correlation between successive cycles, due to the fact that the fission source bank of the previous cycle is used as a source for current cycle. This can severely affect the variance estimation, and always leads to underestimate of the variance. On the other hand, there is a transient phase of the simulation necessary to converge the source distribution to the stationary distribution; during this phase the quantities of interest cannot be tallied and this corresponds to "inactive" cycles. This transient will be evaluated by performing the criticality simulations with two different initial source guess: a uniform shape and a cosine shape, which almost corresponds to the critical shape for our numerical test.

Another method, named the Wielandt's method, has been proposed in order to improve the slow convergence of the power iteration implement. This acceleration method along with the Monte Carlo standard are analysed in terms of the advantages and disadvantages of each in simple but meaningful test cases.

2. BASIC THEORY OF NEUTRON TRANSPORT

The transport equation describes the statistical behaviour of a large population of particles. The exact number of particles per unit volume is continuously varying with time, even at steady state conditions. Under steady state conditions, the number density of particles oscillates around an average value corresponding to the solution of the steady state transport equation. A solution of the transport equation is required in many fields of nuclear engineering, notably in reactor physics, insafety and criticality, and in radiation shielding and protection.

The fundamental assumptions in neutron transport calculation are as following: neutrons ca nbe treated as point-like particles travelling along straight lines between two collision points, and all neutron-neutron interactions can be ignored. These assumptions lead to a linear transport equation. Additional usual assumptions often made are medium homogeneity by regions and time-independent conditions.

2.1. The steady-state Boltzmann neutron transport equation

The stationary behavior of neutral particles transported through matter is described by the linear steady-state transport Boltzmann equation:

$$\hat{\Omega}.\nabla\Psi(r,E,\hat{\Omega}) + \Sigma_t(r,E)\Psi(r,E,\hat{\Omega}) = \int_{4\pi}\int_0^{\infty}\Sigma_s(r,E' \dashrightarrow E,\Omega.\Omega')\Psi\left(r,E,\hat{\Omega}'\right)dE'\,d^2\Omega' + Q(r,E,\hat{\Omega}) \quad (1)$$

where:

- $\Psi(r,E,\hat{\Omega})$ is the angular flux in six-dimensional phase space

- $L\Psi = \hat{\Omega}.\nabla\Psi(r,E,\hat{\Omega})$, the streaming term or leakage term (describe the neutrons crossing the boundaries)

- $T\Psi = \Sigma_t(r,E)\Psi(r,E,\hat{\Omega})$, the collision term

- $S\Psi = \int_{4\pi}\int_0^{\infty}\Sigma_s(r,E' \dashrightarrow E,\Omega.\Omega')\Psi(r,E,\hat{\Omega}')dE'\,d^2\Omega'$, the scattering term (describe the change in energy and direction of neutron particles)

- $M\Psi = Q(r,E,\hat{\Omega})$, the neutron source term

- Σ_s : differential scattering macroscopic cross section

- Σ_t : total macroscopic cross section

For criticality problem, neutron fission source comes from the fissile materials during fission events:

$$Q(r,E,\hat{\Omega}) = \frac{1}{k_{eff}}\frac{\chi(E)}{4\pi}\iint \upsilon\Sigma_f(r,E')\,\Psi\left(r,E',\hat{\Omega}'\right)dE'd^2\Omega' \quad (2)$$

The quantities of interest are:

- the scalar flux distribution or eigenfunction: $\phi(x,E)$

- the largest eigenvalue or multiplication factor: k_{eff}

- various response rates: $R(x) = \int\Sigma(x,E)\Phi(x,E)dE$

2.2. The solution of theneutron transport equation in special case of critical slab geometry

In the case of one-dimensional homogeneous critical slab within one-group diffusion theory, the general form of Eq.(1) can be simplified to the simple form:

$$\frac{d^2\phi(x)}{dx^2} + \chi^2\phi(x) = 0 \quad (3)$$

where $\chi^2 = \frac{(\upsilon\Sigma_f - \Sigma_a)}{D} = \frac{(k_\infty - 1)}{L^2}$ is the material buckling. In general, $k_\infty = \varepsilon p f \eta$ - the infinitive multiplication factor, in which:

- ε: fast fission factor;

- p: resonance escape probability

- f: thermal utilization factor;

- $\eta = \left(\frac{\upsilon\Sigma_f}{\Sigma_a}\right)_{fuel}$: the production factor

By taking into account the condition of the vanishing of the flux at the boundaries and the symmetry of the problem with respect to the plan at x = 0; also the critical condition: $\chi = \frac{\pi}{2a}$, the solution of the Eq.(3) is written as following:

$$\phi(x) = \frac{\pi}{2L}\cos\left(\frac{\pi x}{2L}\right) \quad (4)$$

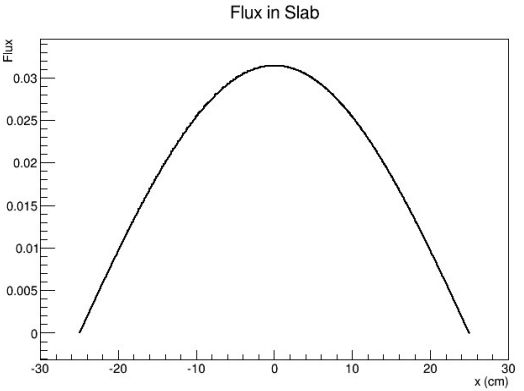

Fig. 1. The distribution of flux inside the critical slab

2.3. The Power iteration method

The standard Monte Carlo criticality calculation uses the fission source iteration that is a simple power iteration to find the fundamental (biggest) eigenvalue k_0 and its associated eigenvector Ψ_0.

Re-writing (1) by using abbreviations:

$$(L + T - S)\Psi = \frac{1}{k_{eff}} M\Psi \qquad (5)$$

The principle of the power iteration method is to evaluate the neutron flux at iteration n+1,$\Psi^{(n+1)}$ by applying the operator $F = (L+T-S)^{-1}M$ on the flux at iteration n,$\Psi^{(n)}$. By supposing that the values of flux and k_{eff} are known from iteration n, we can find out the values for iteration n+1 according to the following procedures:

$$\Psi^{(n+1)} = \frac{1}{k_{eff}^{(n)}} F\Psi^{(n)}$$

$$\text{and} \quad k_{eff}^{(n+1)} = k_{eff}^{(n)} \frac{\int dV\, M\Psi^{(n+1)}}{\int dV\, M\Psi^{(n)}} \qquad (6)$$

where V is the six-dimensional phase space (space, energy and direction).

Supposing $\{u_i\}$ is the set of eigenvectors of the operator F and that it forms a basis, any initial flux distribution can be expanded in function of$\{u_i\}$:

$$\Psi^{(0)} = \sum a_i u_i \text{ with } a_i = \int dV\, \Psi^{(0)} u_i \quad (7)$$

Successive applications of the operator F to the initial distribution will converge to the fundamental eigenvalue and eigenvector. It can be shown that both flux and multiplication factor can be approximated by using [4]:

$$\Psi^{(n+1)} \approx C_1 \left[u_0 + \frac{a_1}{a_0}\left(\frac{k_1}{k_0}\right)^{(n+1)} * u_1\right] \quad \text{and}$$

$$k_{eff}^{(n+1)} \approx k_0 \left[1 + C_2 \frac{a_1}{a_0}\left(\frac{k_1}{k_0}\right)^{(n)} * \left(\frac{k_1}{k_0} - 1\right)\right] \quad (8)$$

It is well-known that the dominant error terms for k_0 decay faster than the dominant error terms for $\Psi^{(0)}$, especially when the dominance ratio $\rho = k_1/k_0$ is close to 1, that is for very large system with characteristic dimensions of several (~100) times the neutron mean free path. For this reason, the fundamental eigenvalue for a high dominance ratio problem will appear to converge faster than the associated eigenvector, which will be more contaminated by the higher harmonics.

3. MONTE CARLO SIMULATIONS

3.1. The random walk for a single neutron

The random walk process provides a faithful simulation of the behaviour of a single particle, from source to death. It is a continuous line made of straight free paths connected at collision locations (scattering, absorption, fission) of the domain and by the cross-section data corresponding to materials present in the domain. During this walk, the particle state is characterized by its position, direction of travel and energy.

The random walks can be generated by:

- sampling the travelled distance in space
- sampling the type of reaction
- sampling the direction of the particle after the collision from the continuous distribution function related to the sampled reaction

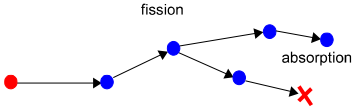

Fig. 2a. Three modes of interaction of neutron

3.2. Criticality calculations

Nuclear criticality is the ability to sustain a chain reaction with fission neutrons. In this case, the Monte Carlo simulation proceeds in cycles, in which the source in each cycle is given by the fission neutron distribution calculated in the previous cycle. Each cycle comprises the simulation of a batch of neutrons containing N source fission neutrons each of them executted by a single random walk. The number of neutrons generated at the end of each cycle is generally not equal to the number ofs ource neutrons which started at the beginning of the cycle. The multiplication factor in cycle n is defined as ratio:

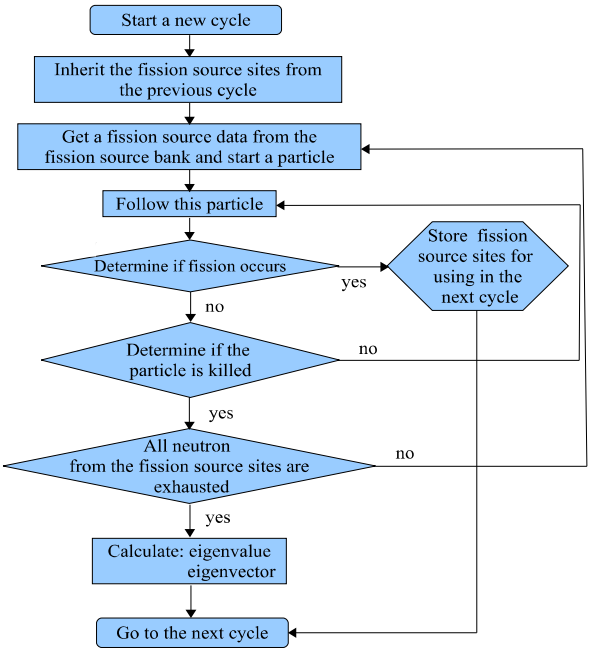

Fig. 2b. Random walk for a single neutron from the source to death

$$k_{eff} = \frac{number\ of\ fission\ neutron\ in\ generation\ N+1}{number\ of\ fission\ neutron\ in\ generation\ N} \quad (9)$$

The quality of the initial spatial distribution of source neutrons is an important issue. A very poor initial source guess can cause several first cycles estimate of k_{eff} to be extremely poor. This situation can occur when only a small fraction of the fission source has a chance to make fissions.

4. ACCELERATION METHODS

4.1. Standard Monte Carlo method

The standard Monte Carlo method for criticality calculation uses the power iteration algorithm to converge the user defined source to the stationary solution. The basis idea of this standard method is to follow neutron particles, from the source to the end of its life by leakage from outer boundaries or absorption, etc, generation by generation. This is achieved by defining an initial fission bank from the user source definition. All the neutrons of the bank are followed one by one until all the particles are exhausted, and whenever a fission occurs the corresponding fission neutrons are put in the next cycle fission bank. This is described in the Fig.3(a,b):

- All fission neutrons of the same generation are put in a cycle
- Fission neutrons of the current cycle become the fission source of the next cycle iteration
- Eigenvalue and eigenvector will be estimated at the end of each cycle.

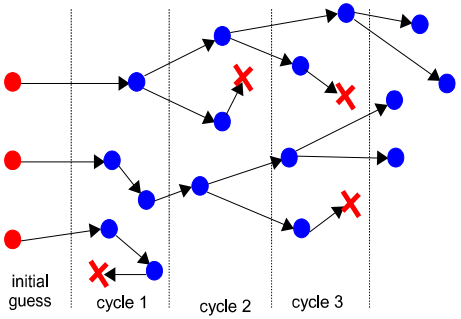

Fig. 3a. Definition the cycle of simulation

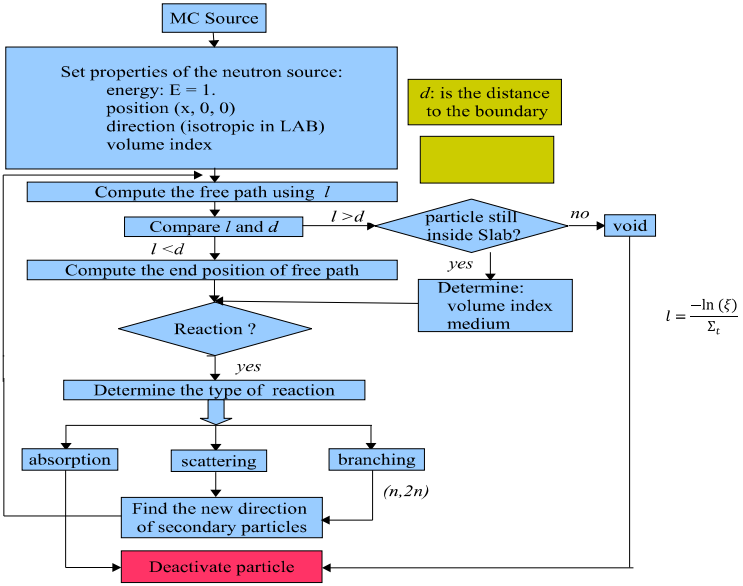

Fig. 3b: Implementation of standard Monte Carlo method

4.2. Wielandt's method for criticality simulation

Wielandt's method was first applied to accelerate the solution of diffusion theory reactor calculation in the 1950s and 1960s. More details of this method were described by Brian et al. (2008). The main principles of criticality simulation by Wielandt's method is described in Fig.4:

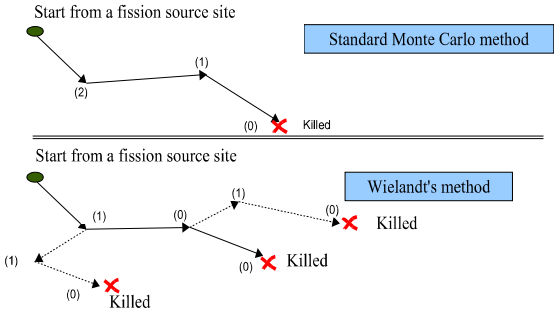

Fig. 4. Compare MC and Wielandt's of implementation

The difference between standard Monte Carlo method and Wielandt's method lies in the way we treat the neutrons produced by fission. For MC, each fission neutron is put in the next cycle, but for the Wielandt's method, a fraction of these neutrons are kept in the current cycle and continued to be tracked in a single random walk by the probability condition:

$$N_{current} = \frac{\nu\Sigma_f}{\Sigma_t}\left(\frac{1}{k_{eff}} - \frac{1}{k_e}\right) + \xi \leq 0{,}8 \quad (10)$$

$k_e = k_{eff} + 4\sigma(\sigma(k_{eff}))$; σ is the variance of variance of k_{eff}

If fission is determined to be occurring, the number of fission neutrons $N_{current}$ are tracked by the same random walks in the current cycle just like other source particles. These neutrons continue producing their progenies, and continue tracking within the current cycle until their death by leakage, absorption, etc, (Fig. 4). The chain of current-generation neutrons followed by such away will be treated as a part of the same initial history in the iteration, so that correlation effects between successive fissions will decrease. This improved statistical treatment should reduce the under-prediction bias in confidence intervals for criticality calculations. The value on the right hand side of Eq.(10) depends on the size of the slab, the larger the slab is the higher the value is.

5. RESULTS AND DISCUSSION

5.1. Numerical parameters

5.1.1. Properties of homogeneous slab

To simplify our problem, we consider a 1D homogeneous slab with void boundary conditions. In our criticality simulations, the particles are supposed to be mono-kinetic (MKParticle: its velocity is always constant and it doesn't change with scattering collisions), and the fission and scattering interactions are supposed isotropic. The homogeneous medium was chosen with the following properties in Tab. 1.

5.1.2. Parameters of simulations

- Simulations will be performed for three different lengths of slab: L = 10; 50; 100 (cm)

- Two different shapes of initial fission source guess (uniform and cosine shape) are also investigated to analyze the behavior of the transient phase.

Tab. 1. Macroscopic cross section for homogeneous slab

Types of reaction	total	absorption	scattering	branching	$\bar{\nu}$
Macroscopic cross section (cm^{-1})	$\Sigma_t = 1{,}0$	$\Sigma_a = 0{,}2$	$\Sigma_s = 0{,}8$	$\Sigma_f = 0{,}1$	2

5.2. Numerical results

5.2.1. The dependence of k_{eff} on the size of the critical slab

By definition of multiplication factor, this quantity is the ratio of the total number of particles produced by fission at the end of the cycle to the number of initial particles; this ratio is strongly sensitive to the effect of leakage from the boundaries. For the same slab material properties, the smaller the length of the slab, the bigger the effect of leakage, and finally the lower the value of k_{eff}. We also note that the smaller the size of the slab (in mean free paths), the better the particle can explore the entire slab in a single generation. For large slabs, it takes several generations for a particle to travel from one side of the system to the other. Note that: 1pcm = 10^{-5}

The adoption of the Wielandt's method leads to an increase of statistics because of the tracking of additional particles in each cycle. Consequently, the fluctuations and the standard deviation of multiplication factor are less than the one implemented by conventional Monte Carlo simulations. Those results could be seen clearly on Fig. 5 for the case L = 100 cm.

5.2.2. Figure of Merit (FOM)

In Monte Carlo simulations, the variance associated with the estimated quantities is (asymptotically) linearly proportional to the inverse of the number of histories simulated: $\sigma^2 = {}^1/_{\sqrt{N}}$. On the other hand, the simulation time is roughly proportional to the number of histories. This has led, for the comparison of different simulations and/or different methods in terms of efficiency, to the definition of a criterion called Figure of Merit (FOM): $FOM = \frac{1}{\sigma^2 T}$ (11), in which T is the simulation time.

When comparing two set of simulations on the same problem, the run with a bigger FOM is considered more efficient (smaller variance and/or smaller computing time). To evaluate the efficiency of our different implemented methods, we have carried out a group of 1000 independent simulations, 1000 particles, 1000 cycles for slab length L = 50 cm.

The FOM for MC and Wielandt's are not much different and the gains in variance due to the improved statistics are compensated by the extra time spent in the simulations (Table 3).

Tab. 2. The values of k_{eff} for different methods

Slab length (L in cm)	MC (σ *in pcm*)	Wielandt (σ *in pcm*)
10	0,88931 ± 3,2	0,88845 ± 1,5
50	0,99371 ± 20,1	0,99298 ± 5,6
100	0,99832 ± 28,9	0,99779 ± 16,6

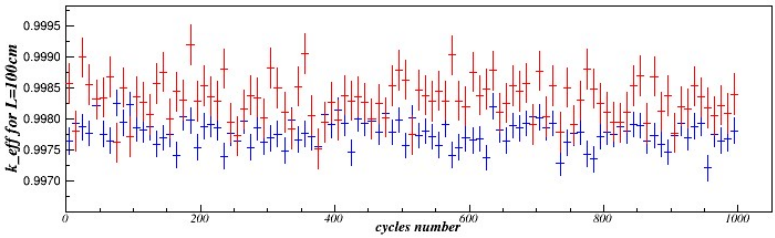

Fig. 5. The fluctuation k_{eff} in MC and Wielandt for L = 100 cm

Tab. 3. FOM of 1000 independent simulationsfor L = 50 cm

Methods	MC	Wielandt's
Time (h)	100	419
$\sigma^2(10^{-10})$	404	87
FOM	69	77

5.2.3. Auto-correlation estimation

The cycle to cycle correlation is one of the main problems of the power iteration method, due to the fact that the source of neutrons for the next cycle comes from neutron which have suffered fission in the current cycle. This correlation depends on the size of the system: more cycles are required for neutron to be propagated from one side to the other of larger systems. This correlation length (Malvagi et al., 2012) can be estimated at the end of the Monte Carlo simulation by introducing an empirical correlation function as following:

$$C_k = \frac{\langle(\Phi_i-\mu)*(\Phi_{i+k}-\mu)\rangle}{\sigma^2} \qquad (12)$$

where:

- Φ_i the flux at cycle *i;*
- μ the expected value of variable Φ
- σ^2 the variance of variable Φ;
- $k \geq 1$

A low value of the auto-correlation indicates that the sequence made by consecutive values of Φ_i with $i = 1,2 ,\dots , k$ are weakly correlated. We see that the auto-correlation decreases with the value of the lag k (Fig.6). When this value is below the cutoff threshold of 0.2, we empirically consider that is uncorrelated. On Fig.7 we plot the value of the lag for which the auto-correlation descends under the cutoff versus the bin number corresponding to the slab with the length of 100 and 50 cm, respectively, simulated by standard MC and Wielandt's method. This figure is consistent with the theory discussed above: the larger the slab, the higher the value of correlation length. By adopting Wielandt's method to treat the cycle-to-cyclecorrelation, this quantity strongly decreases from 86 to 45 for L = 100 cm (high DR problem) and from 40 to 10 for L = 50 cm. From our simulations for L = 10 cm, there is no correlation problem for any of the methods. It also remarks that the correlations are at the lowest near the boundaries of the system (this is due to the effects of the boundary conditions) and at its center (this is where the second harmonics has a node).

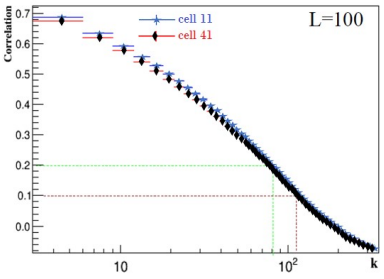

Fig. 6. Correlation vs k for bin 11 (start curve) and bin 41 (diamon curve) for L = 100

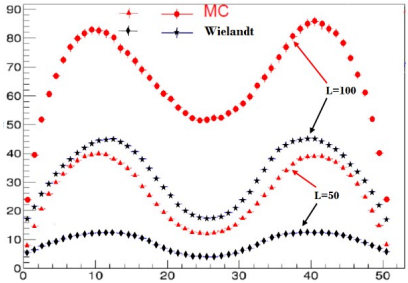

Fig. 7. Correlation cut-off (0,2) as function of number of bin cells

5.2.4. Wielandt's method implementation to cancel the auto-correlation

The basic idea of Wielandt's method is to keep neutrons to stay longer in each cycle and, thus, spreading out more in space compared to the standard MC. To be better understood the effects of this method on the correlation lengths, we carried out the simulations for L = 100 cm (large size) for a set of 100 independent simulations, varying the fraction of fission neutrons kept in each cycle.

Fig. 8 plotts the max of correlation cutoff in the slab as a function of the fraction of fission neutrons kept in each cycle. We can conclude that for high dominant ratio systems (larger size), the Wielandt's method is a useful way to decrease the cycle by cycle correlations.

5.3. Fission source convergence

5.3.1. Characteristics of the fission source convergence as a function of the initial source guess and slab size

The power iteration technique allows to converge a user defined source to the critical source distribution in Monte Carlo simulation. For 1D homogeneous slab geometry, void boundary condition and for both uniform and cosine initial source guess distribution, we expect the equilibrium flux distribution to assume a cosine form in space (predicted by diffusion theory). The convergence of the criticality simulation is estimated by measuring the flux in each spatial bin. In this part, the behaviour of fission source convergence depending on the slab length and initial source guess is investigated. The results are described in Fig. 9.

The Fig. 9(a) shows the flux convergence and fluctuations, for L = 50 cm, for the both uniform and cosine shape implemented by standard MC and Wielandt's for simulations. These indicate the importance of choosing the right initial source guess for MC to quickly converge the flux distribution. However, by adopting Wielandt's method, we can find the robust initial guess of user defined source because of increasing the statistics of each cycle iteration leading to increase the spatial distribution.

For the case of L = 100 cm**,** in Fig.9(b)**,** the results are presented for the same uniform source guess but for the set of 1 and 100 independent simulations. The fluctuations and, specially, transient phase are hightly enhanced. We obtain very large size system with a high dominance ratio.

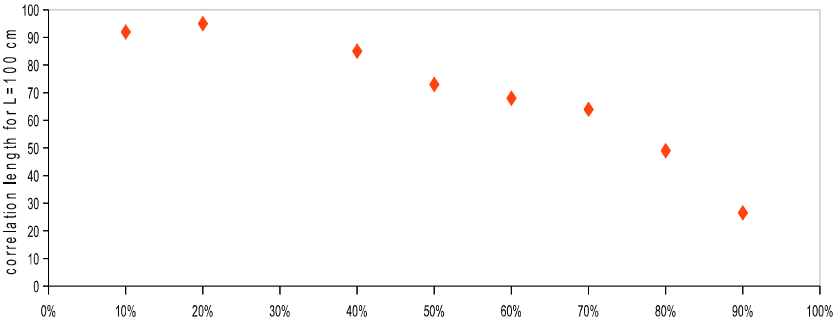

Fig. 8. Correlation length for cut-off factor 0,2 and L = 100 cm

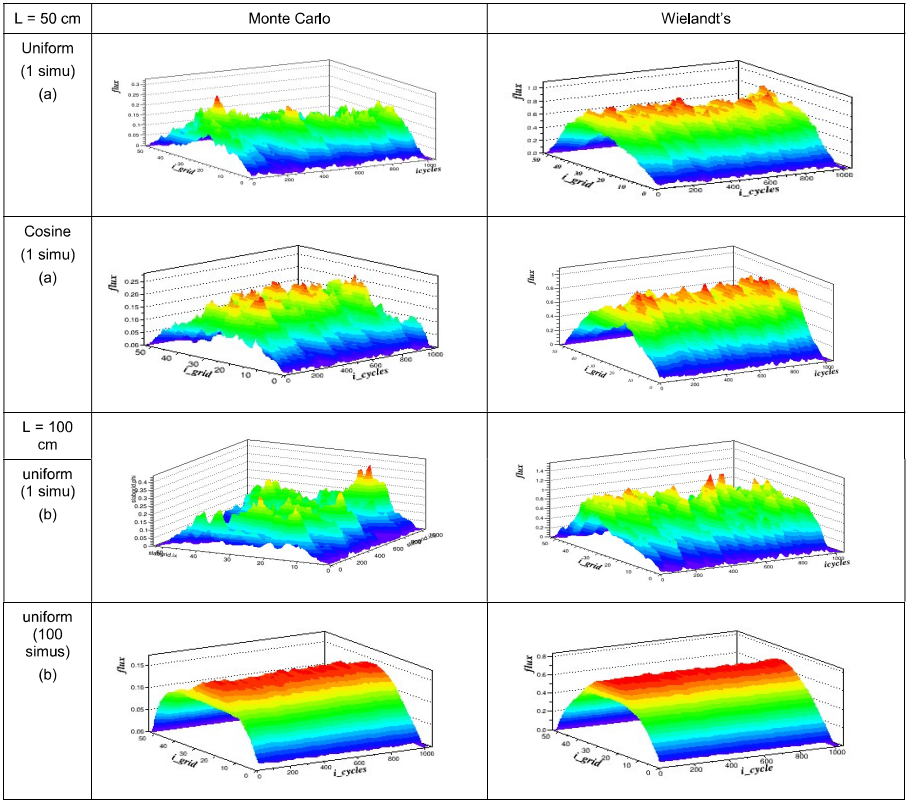

Fig. 9. The distribution of flux in special grid as function of cycle iteration

For each independent simulation, standard MC results in the source distribution are very concentrated in some small region of the slab. This behavior can be explained as follow: in the regions where more fission neutrons exist, there will be a high probability for the neutrons interacting with medium to produce new fission neutrons. On the contrary, by adopting Wielandt's method for the same initial number of particles: moreparticles will be tracked in one cycles; these particles will expand in a greater spatial region of the slab system. Consequently, the fission source distribution will spread out more in a given number of cycles.

6. CONCLUSION AND FUTURE OUTLOOK

The main objective of this article was to use a Python prototype to implement the standard Monte Carlo criticality power iterations for mono-kinetic particles and to compare with the Wielandt's method of accelerating the power iteration. Numerical simulations of a critical slab of varying lengths where then used to compare the performances of these methods.

The author was successful to implement and modify the available source code in the laboratory of SERMA in CEA for the standard Monte Carlo which is the most basic method of

implementation to describe the interaction of neutrons in the core. However, due to the slow convergence of the source, especially in the case of the linear initial shape and large size of the geometry, it needs to accelerate the process of the convergence to get the better results in the same level of the competence.

The Wieland's method implemented was only concentrated in the direction of accelerating the source convergence and there were no ideas to estimate the multiplication factor, k_{eff}, which is one of the most important quantity to know if the new method is suitable or not.

The three keys objectives has been obtained as following:

- Based on the basic of Wielandt's method to accelerate and to reduce the time of convergence of any kind of initial guess source.

- Proposing the new way to estimate the value of k_{eff} inside the code and compare it to the one of MC

- Estimating the correlation between cycle to cycle by increasing the statistics for the same initial condition with the old method of MC.

There is a need of future work for a better idea of the biases of those methods that can introduce into the simulation as a function of their respective parameters: fraction of fission neutrons kept in the current cycle for Wielandt's. Another question not explored is the optimisation between the number of cycles run in a simulation and the number of independent simulations in order to minimize the final variance, which will certainly depend on the method.

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